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AN ELASTIC STRIP MOVING ACROSS A RIGID STEP

G. G. ADAMS
Department of Mechanical Engineering, Northeastern University, Boston, MA 02115, U.S.A.

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Abstract—An infinite elastic strip moves at constant speed across a frictionless rigid foundation possessing a step discontinuity. Transform methods are used to reduce the mixed boundary value problem to a system of singular integral equations. For a range of step height and strip speed, the noncontact regions, lower boundary displacements, and foundation contact pressures are determined. The results show that different types of solutions exist for given combinations of speed and height. The solution for the corresponding problem of a stationary strip are also given.

INTRODUCTION

The response of elastic beams and strips to moving loads has been the object of many investigations. One class of problems occurs when the beam or strip has a one-sided constraint. The first study of such a problem was made by Adams and Bogy[1] who solve the problem of an elastic beam resting on a rigid foundation and subjected to a moving load. Both Euler–Bernoulli and Timoshenko beam models were used. In [2] and [3] Adams solves related problems using plane strain elasticity theory.

An elastic foundation which acts in compression only can be considered a generalization of an unilateral constraint (rigid foundation). In [4] and [5] Adams determines the response of an elastic strip pressed against an elastic half plane by a steadily moving load. One characteristic of the solutions in [1–5] is that for given force and speed more than one type of solution is possible, i.e. the solution is not unique. This point will be discussed later.

In the present investigation we obtain the solution for an infinite elastic strip moving with constant speed across a frictionless and rigid foundation which possesses a step discontinuity. A similar problem involving an elastic beam is solved in [6]. In the present problem, a solution is obtained by applying Fourier transforms to the equations of motion of a steadily moving elastic strip (plane strain). By choosing an appropriate integral representation, we reduce the mixed boundary value problem to a system of singular integral equations which are solved numerically. The results so obtained are the noncontact regions, lower boundary displacements, and foundation reaction pressures. The solution for a stationary strip is also presented.

PROBLEM FORMULATION

A uniform, isotropic and homogeneous elastic strip of constant thickness "a" moves to the right with constant speed "c" across a frictionless and rigid base (Fig. 1). The foundation possesses the step discontinuity of magnitude 2 h which tends to produce a partial separation of the layer from the foundation. Using the two-dimensional (plane strain) theory of elasticity, we write the displacement equations of motion of the strip as

$$\mu\nabla^2\bar{u}_\alpha + (\lambda+\mu)\frac{\partial\bar{v}}{\partial x_\alpha} = \rho\frac{\partial^2\bar{u}_\alpha}{\partial t^2} - F_\alpha, \quad \alpha = 1,2 \tag{1}$$

where λ, μ, ρ are the Lamé's constant, shear modulus, and mass density, $\bar{u}_\alpha$, F_α are the components of displacement, and body force, and where

$$\bar{v} = \frac{\partial\bar{u}_1}{\partial x_1} + \frac{\partial\bar{u}_2}{\partial x_2}, \quad F_\alpha = (0, -\rho g).$$

Rewriting eqn (1) in terms of a dimensionless coordinate system, fixed in space with respect to the stationary step, we obtain

$$(\delta^2-\beta_1^2)\frac{\partial^2 u_1}{\partial\xi_1^2}+\frac{\partial^2 u_1}{\partial\xi_2^2}+(\delta^2-1)\frac{\partial^2 u_2}{\partial\xi_1\partial\xi_2}=0,$$

$$\delta^2\frac{\partial^2 u_2}{\partial\xi_2^2}+(1-\beta_2^2)\frac{\partial^2 u_2}{\partial\xi_1^2}+(\delta^2-1)\frac{\partial^2 u_1}{\partial\xi_1\partial\xi_2}=0, \tag{2}$$

where

$$\xi_1=(x_1-ct)/a,\quad \xi_2=x_2/a,$$
$$u_\alpha^*(\xi_1,\xi_2)=(\mu/\rho g a^2)\bar{u}_\alpha(x_1,x_2,t),\quad h=(\mu/\rho g a^2)\bar{h}$$
$$u_\alpha=u_\alpha^*-u_\alpha^g,\quad u_1^g=0,\quad u_2^g=-\xi_2(1-\xi_2/2)/\delta^2$$
$$\beta_\alpha=c/c_\alpha,\quad \delta=c_1/c_2,\quad c_1=\sqrt{[(\lambda+2\mu)/\rho]},\quad c_2=\sqrt{(\mu/\rho)}. \tag{3}$$

We note that u_α^* is the dimensionless displacement, u_α^g is the dimensionless displacement due to gravity, and u_α is a residual displacement field.

The boundary conditions to be applied are of the mixed type and are given by

$$\sigma_{12}(\xi_1,0)=\sigma_{12}(\xi_1,1)=0,\quad \xi_1\in\mathscr{R} \tag{4}$$

$$\sigma_{22}(\xi_1,1)=0,\quad \xi_1\in\mathscr{R} \tag{5}$$

$$u_2(\xi_1,0)=\operatorname{sgn}(\xi_1)h,\quad \xi_1\in\mathscr{R}-\Omega, \tag{6}$$

$$\sigma_{22}(\xi_1,0)=1,\quad \xi_1\in\Omega, \tag{7}$$

where $\mathscr{R}$ is the real line $(-\infty,\infty)$ and Ω is the region(s) of separation of the layer from the foundation (initially unknown). Note that (7) is a residual stress condition. The extra conditions required to determine the noncontact region(s) Ω are that the slope of the bottom surface of the elastic strip be continuous at all contact points excluding the step corner ($\xi_1=0$). In addition, any solution satisfying (2)–(7) and the regularity conditions must satisfy the following inequalities:

$$\sigma_{22}^*(\xi_1,0)<0,\quad \xi_1\in\mathscr{R}-\Omega, \tag{8}$$

$$u_2^*(\xi_1,0)>\operatorname{sgn}(\xi_1)h,\quad \xi_1\in\Omega. \tag{9}$$

These conditions state that the normal contact stress remain compressive in the contact region and that the lower boundary normal displacement not interfere with the geometry of the foundation.

METHOD OF SOLUTION

Proceeding in a manner similar to [2], we apply the exponential Fourier transform, with respect to x_1, to (2), (4), (5). The resulting integral representations of the displacement

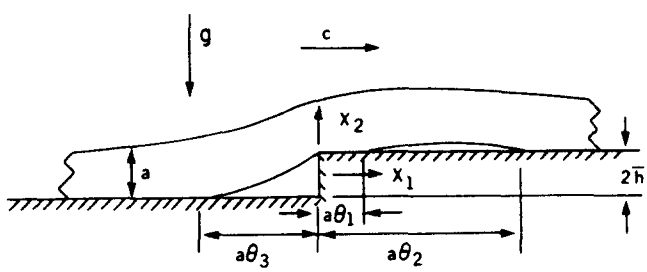

Fig. 1. An infinite elastic strip moving with constant speed across a rigid step.

components are given in [2] with $P=0$. We must still satisfy the mixed conditions (6), (7) as well as the regularity conditions. Evaluating the results of [2], we obtain the normal displacement and normal stress on the lower boundary of the strip as

$$u_2(\xi_1,0)=\frac{i\beta_2^2}{2\sqrt{(2\pi)}}\int_{-\infty}^{\infty}\omega B(\omega)\,e^{-i\omega\xi_1}\,d\omega \tag{10}$$

$$\sigma_{22}(\xi_1,0)=\frac{i}{\sqrt{(2\pi)}}\int_{-\infty}^{\infty}\omega B(\omega)R(\omega)\,e^{-i\omega\xi_1}\,d\omega \tag{11}$$

where

$$\begin{aligned}R(\omega)&=2\lambda_2[2\alpha(1-\cosh\lambda_1\cosh\lambda_2)\\&\quad+(\alpha^2+1)\sinh\lambda_1\sinh\lambda_2]/\Delta(\omega),\\ \Delta(\omega)&=\alpha\cosh\lambda_1\sinh\lambda_2-\cosh\lambda_2\sinh\lambda_1,\\ \alpha&=(1-\tfrac{1}{2}\beta_2^2)^2/\kappa_1\kappa_2,\quad \lambda_\gamma=\omega\kappa_\gamma,\ \kappa_\gamma=(1-\beta\gamma^2)^{\frac{1}{2}},\quad \gamma=1,2,\end{aligned} \tag{12}$$

and $B(\omega)$ is unknown.

In order to satisfy (6), we decompose $B(\omega)$ into its even and odd parts

$$\omega B(\omega)=B_1(\omega)+B_2(\omega),$$

where

$$B_1(\omega)=-B_1(-\omega),\quad B_2(\omega)=B_2(-\omega) \tag{13}$$

and make use of the following integral representations

$$\begin{aligned}B_1(\omega)&=\frac{\sqrt{(2\pi)}}{\omega\beta_2^2}\int_\Omega\phi(t)\cos\omega t\,dt,\\ B_2(\omega)&=\frac{i\sqrt{(2\pi)}}{\omega\beta_2^2}\int_\Omega\phi(t)\sin\omega t\,dt.\end{aligned} \tag{14}$$

The lower boundary displacement finally becomes

$$\begin{aligned}u_2(\xi_1,0)&=\pi\int_\Omega\phi(t)[H(\xi_1)H(t)-H(-\xi_1)H(-t)]H(|t|-|\xi_1|)\,dt\\&\quad-(\pi/2)\,\mathrm{sgn}\,(\xi_1)\int_\Omega\phi(t)\,dt,\quad -\infty<\xi_1<\infty\end{aligned} \tag{15}$$

where the identity which can be obtained from [7; p. 18, eq. 1]

$$\int_0^\infty p^{-1}\sin px\cos py\,dp=(\pi/2)\,\mathrm{sgn}\,(x)H(|x|-|y|)$$

was used with $H(x)$ the unit step function.

Substituting (13), (14) into (11), using the symmetry of $R(\omega)$ and reversing the order of integration we obtain

$$\sigma_{22}(\xi_1,0)=(2/\beta_2^2)\int_\Omega\phi(t)\int_0^\infty\omega^{-1}R(\omega)\sin\omega(t-\xi_1)\,d\omega\,dt \tag{16}$$

for the normal stress on the lower boundary of the strip. By decomposing $R(\omega)$ according to

$$R(\omega)=-2\kappa_2(1-\alpha)[1-k(\omega)],$$

where

$$k(\omega) = -\{\alpha^2 e^{-\lambda_1}\sinh\lambda_2 + \alpha[e^{-\lambda_1}\cosh\lambda_2 + e^{-\lambda_2}\cosh\lambda_1 - 2] + e^{-\lambda_2}\sinh\lambda_1\}/(1-\alpha)\Delta(\omega) \tag{17}$$

and using

$$\frac{\mathrm{d}}{\mathrm{d}y}\int_0^\infty p^{-1}\sin px\sin py\,\mathrm{d}p = \frac{x}{x^2-y^2},$$

which can be obtained by differentiating [7, p. 78, eq. 1], the foundation contact pressure becomes

$$r^*(\xi_1) = 1 + 2\eta\int_\Omega\left[\frac{1}{t-\xi_1} + K(\xi_1 - t)\right]\phi(t)\,\mathrm{d}t, \quad -\infty<\xi_1<\infty \tag{18}$$

where

$$K(x) = \int_0^\infty k(\omega)\sin\omega x\,\mathrm{d}\omega,$$
$$\eta = 2\kappa_2(1-\alpha)/\beta_2^{\,2} \tag{19}$$

and

$$r^*(\xi_1) = 1 - \sigma_{22}(\xi_1, 0). \tag{20}$$

ONE REGION OF NONCONTACT

For sufficiently small combinations of step height and speed, we may expect the strip to separate from the foundation in only one region immediately to the left of the step corner. Setting the noncontact region

$$\Omega = \{\xi_1 | -\theta_3 < \xi_1 < 0\} \tag{21}$$

and applying (21) to (15) we obtain

$$u_2(\xi_1, 0) = \begin{cases} -h, & -\infty<\xi_1<-\theta_3 \\ -\pi\displaystyle\int_{-\theta_3}^{\xi_1}\phi(t)\,\mathrm{d}t - h, & -\theta_3<\xi_1<0 \\ h, & 0<\xi_1<\infty \end{cases} \tag{22}$$

provided

$$\int_{-\theta_3}^{0}\phi(t)\,\mathrm{d}t = -\frac{2h}{\pi} \tag{23}$$

where (22) satisfies (6). We must now satisfy the mixed normal stress condition (7), which, using (18), (20), (21) becomes

$$2\eta\int_{-\theta_3}^{0}\left[\frac{1}{t-\xi_1} + K(\xi_1 - t)\right]\phi(t)\,\mathrm{d}t = -1, \quad -\theta_3<\xi_1<0. \tag{24}$$

The singular integral equation (24) is also subject to (23) as well as the regularity condition at $\xi_1 = -\theta_3$

$$\lim_{\epsilon\to 0}\frac{\mathrm{d}u_2}{\mathrm{d}\xi_1}(-\theta_3+\epsilon, 0) = 0. \tag{25}$$

Due to the presence of the step corner, there is no regularity condition at the point $\xi_1 = 0$; the normal stress will be singular and the lower boundary displacement will be discontinuous at this point.

TWO REGIONS OF NONCONTACT

Solutions obtained with only one region of noncontact are valid provided the inequality constraints (8) and (9) are satisfied. Solutions for two regions of noncontact can be obtained by using

$$\Omega = \{\xi_1 | -\theta_3 < \xi_1 < 0, \quad \theta_1 < \xi_1 < \theta_2\} \tag{26}$$

which together with (15) results in

$$u(\xi_1, 0) = \begin{cases} -h, & -\infty < \xi_1 < -\theta_3 \\ -\pi \int_{-\theta_3}^{\xi_1} \phi_1(t)\,\mathrm{d}t - h, & -\theta_3 < \xi_1 < 0 \\ h, & 0 < \xi_1 < \theta_1 \\ -\pi \int_{\theta_1}^{\xi_1} \phi_2(t)\,\mathrm{d}t + h, & \theta_1 < \xi_1 < \theta_2 \\ h, & \theta_2 < \xi_1 < \infty \end{cases} \tag{27}$$

provided

$$\int_{-\theta_3}^{0} \phi_1(t)\,\mathrm{d}t = -\frac{2h}{\pi}, \quad \int_{\theta_1}^{\theta_2} \phi_2(t)\,\mathrm{d}t = 0. \tag{28}$$

The system of two singular integral equations for determining the unknown functions $\phi_1(t)$ and $\phi_2(t)$ is

$$\begin{aligned} &2\eta \int_{-\theta_3}^{0} \left[\frac{1}{t-\xi_1} + K(\xi_1 - t)\right]\phi_1(t)\,\mathrm{d}t \\ &+ 2\eta \int_{\theta_1}^{\theta_2} \left[\frac{1}{t-\xi_1} + K(\xi_1 - t)\right]\phi_2(t)\,\mathrm{d}t = -1, \\ &-\theta_3 < \xi_1 < 0, \quad \theta_1 < \xi_1 < \theta_2, \end{aligned} \tag{29}$$

which is subject to the conditions given in (28), and regularity conditions of smooth contact of the form of (25) at the points $\xi_1 = -\theta_3$, $\xi_1 = \theta_1$, $\xi_1 = \theta_2$. These conditions will be incorporated directly into the numerical procedure described below.

STATIONARY STRIP

The solution for a stationary strip can be obtained either by taking an asymptotic expansion for small speeds, or by resolving the problem. For brevity we simply list the results

$$\eta = \tfrac{1}{2}(1-\nu)$$

$$k(\omega) = [2\omega(\omega+1) + 1 - e^{-2\omega}]/(\sinh 2\omega + 2\omega).$$

Using (18), $(19)_1$, (21)–(25) solutions may be obtained for one noncontact region. Solutions for two noncontact regions may then be determined from (18), $(19)_1$, (26)–(29).

NUMERICAL SOLUTIONS

We will now solve the system of singular integral equations previously defined using the collocation method of Erdogan and Gupta[8]. Focusing our attention on the solution of the equations (27) and (28), corresponding to two noncontact regions we make the following linear transformations

$$s_i = \frac{2t}{(b_i - a_i)} - \frac{(b_i + a_i)}{(b_i - a_i)}, \quad r_i = \frac{2\xi_1}{(d_i - c_i)} - \frac{(d_i + c_i)}{(d_i - c_i)}, \quad i = 1, 2 \tag{30}$$

where

$$\begin{aligned} &a_1 = \theta_1, && b_1 = \theta_2, && \theta_1 < t < \theta_2 \\ &a_2 = -\theta_3, && b_2 = 0, && -\theta_3 < t < 0 \\ &c_1 = \theta_1, && d_1 = \theta_2, && \theta_1 < \xi_1 < \theta_2 \\ &c_2 = -\theta_3, && d_2 = 0, && -\theta_3 < \xi_1 < 0. \end{aligned}$$

Then defining

$$\Psi_i(s_i) = \phi_i(t), \quad i = 1, 2$$

$$\bar{\Psi}_1(s_1) = \Psi_1(s_1)\sqrt{[(1 - s_1)/(1 + s_1)]}, \quad \bar{\Psi}_2(s_2) = \Psi_2(s_2)/\sqrt{(1 - s_2^2)} \tag{31}$$

eqns (27) and (28) are approximated[8] by the following system of $2N + 3$ linear algebraic equations

$$2\eta \sum_{i=1}^{2} \sum_{J=1}^{N} A_{iJ} \bar{\Psi}_i(s_{iJ}) \left[\frac{1}{t_{iJ} - \xi_{1iJ}} + K(\xi_{1iK} - t_{iJ}) \right] l_i = -1,$$

$$K = 1, 2, \ldots, \tilde{N}, \quad \tilde{N} = N + i - 1$$

$$\sum_{J=1}^{N} A_{iJ} \bar{\Psi}_i(s_{iJ}) = 0, \quad i = 1, 2 \tag{32}$$

where

$$t_{iJ} = \frac{b_i - a_i}{2} s_{iJ} + \frac{b_i + a_i}{2}, \quad J = 1, 2, \ldots, N, \quad i = 1, 2$$

$$\xi_{1iK} = \frac{d_i - c_i}{2} r_{iK} + \frac{d_i + c_i}{2}, \quad K = 1, 2, \ldots, \tilde{N}, \quad i = 1, 2$$

$$l_i = \frac{b_i - a_i}{2}, \quad i = 1, 2,$$

$$A_{1J} = \frac{2\pi(1 + s_{1J})}{2N + 1}, \quad A_{2J} = \frac{\pi}{N + 1}(1 - s_{2J}^2)^{\frac{1}{2}}, \quad J = 1, 2, \ldots, N$$

$$s_{1J} = \cos\left(\frac{2(J - 1)\pi}{2N + 1}\right), \quad s_{2J} = \cos\left(\frac{J\pi}{N + 1}\right), \quad J = 1, 2, \ldots, N$$

$$r_{1K} = \cos\left(\frac{2K\pi}{2N + 1}\right), \quad r_{2K} = \cos\left(\frac{\pi(2K - 1)}{2N + 2}\right), \quad K = 1, 2, \ldots, N + 1.$$

The system of eqns (32) is linear in the $2N$ unknowns $\bar{\Psi}_i(s_{iJ})$ but nonlinear in θ_1, θ_2 and θ_3. However, since the system is linear in h, we may treat (32) as if h were unknown and θ_3 known. This leaves us with a system of $2N + 3$ equations which is linear in $2N + 1$ unknowns and nonlinear in θ_1 and θ_2 which can be obtained by a simple iterative process. Having solved (32), we can now determine the contact pressure by a simple quadrature of (18), while the lower boundary displacement may be determined from (27).

Due to the singularity at the step corner we define the stress intensity factor K by

$$K = \lim_{\xi_1 \to 0} \xi_1^{\frac{1}{2}} r^*(\xi_1),$$

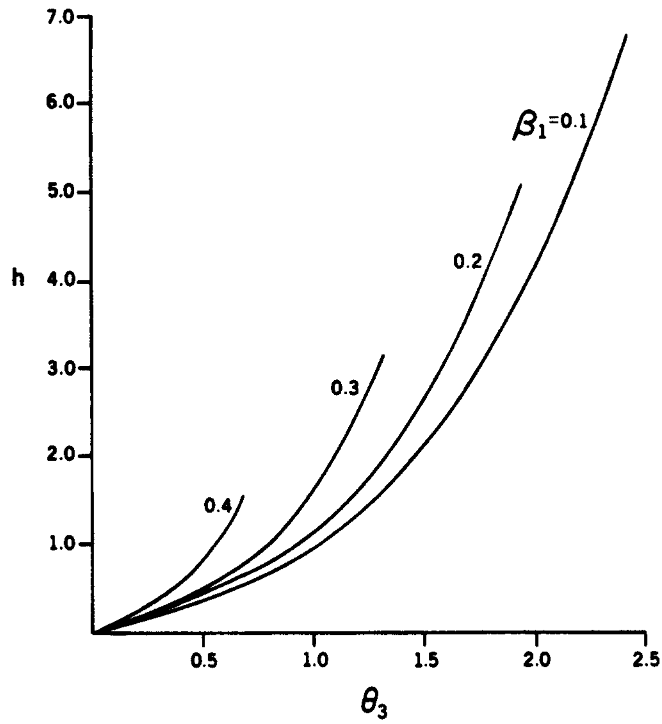

Fig. 2. Step height (h) vs noncontact length (θ_3) for one noncontact region and various values of strip speed (β_1).

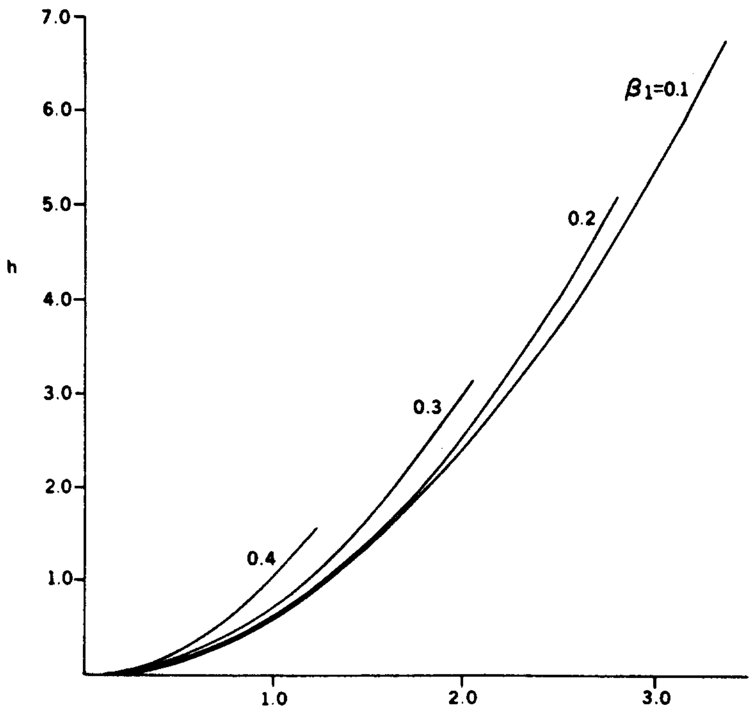

Fig. 3. Step height (h) vs stress-intensity factor (K) for one noncontact region and various values of strip speed (β_1).

which can be expressed as

$$K = -2\theta_1^{\frac{1}{2}}\pi\eta\bar{\Psi}_1(1) \tag{33}$$

where $\bar{\Psi}_1(1)$ must be determined by interpolation.

RESULTS AND DISCUSSION

A graph of step height (h) vs noncontact length (θ_3) corresponding to one noncontact region is given in Fig. 2 for various speeds (β_1).† Note that θ_3 increases monotonically with h and that for given step height the value of θ_3 is greater for lower speeds. Figure 3 shows step height (h) vs stress-intensity factor (K) again for one noncontact region and various speeds (β_1). Note that for given h the value of K is greater at lower speeds. This is because at lower values of β_1 the corresponding value of θ_3 is higher (for given h) and hence the step corner carries a greater

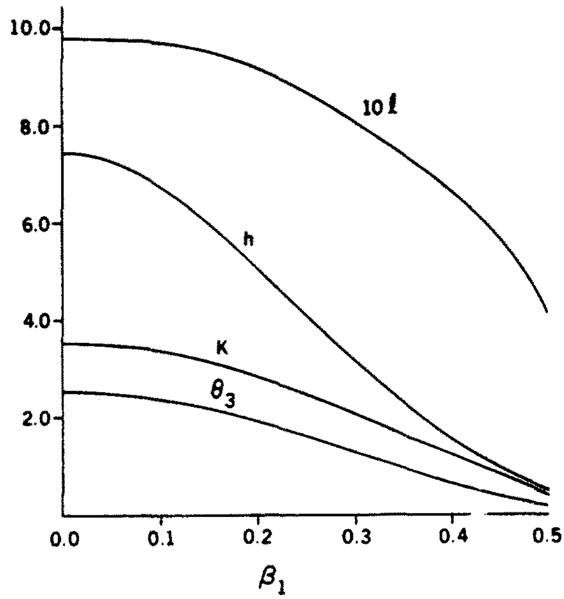

Fig. 4. Limiting values of noncontact length (θ_3), stress-intensity factor (K), step height (h), and lift-off location (l) vs strip speed (β_1) for one noncontact region.

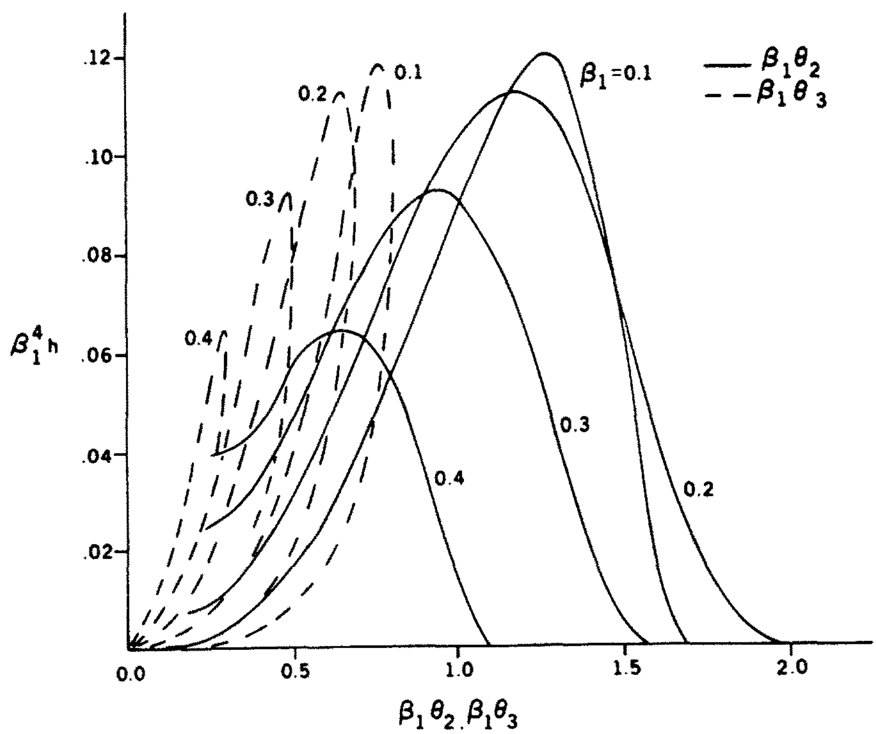

Fig. 5. Step height vs noncontact lengths for two noncontact regions and various speeds.

†All results given in this paper are for a value of 0.25 for Poisson's ratio. Furthermore we emphasize that "h" is a dimensionless quantity as defined by (3).

load. For a specified value of θ_3 the value of the stress-intensity factor does increase with speed. In Fig. 4 we show a graph of the limiting values of step height (h), stress-intensity factor (K), noncontact length (θ_3), and location of the zero pressure point (l) vs speed β_1 at which a second noncontact region initially appears. All of these quantities decrease monotonically with speed (β_1).

In Fig. 5 we show a quantity related to step height ($\beta_1{}^4h$) vs noncontact lengths ($\beta_1\theta_2$, $\beta_1\theta_3$) for the first mode corresponding to two noncontact regions and various speeds (β_1). Note that the behavior is no longer monotonic. At any speed, as θ_2 increases from its initial value (l in Fig. 4), h increases until it reaches a maximum and then decreases to zero. Similarly θ_3 increases to a maximum and then decreases to zero. With both θ_3 and h equal to zero we have the first "free mode" solutions given in [2] for a flat foundation and zero load. We also observe that as speed increases, the maximum value of h for this first mode also increases. A graph of $\beta_1{}^4h$ vs $\beta_1{}^2K$ for various speeds is given in Fig. 6. At given speed larger values of step height

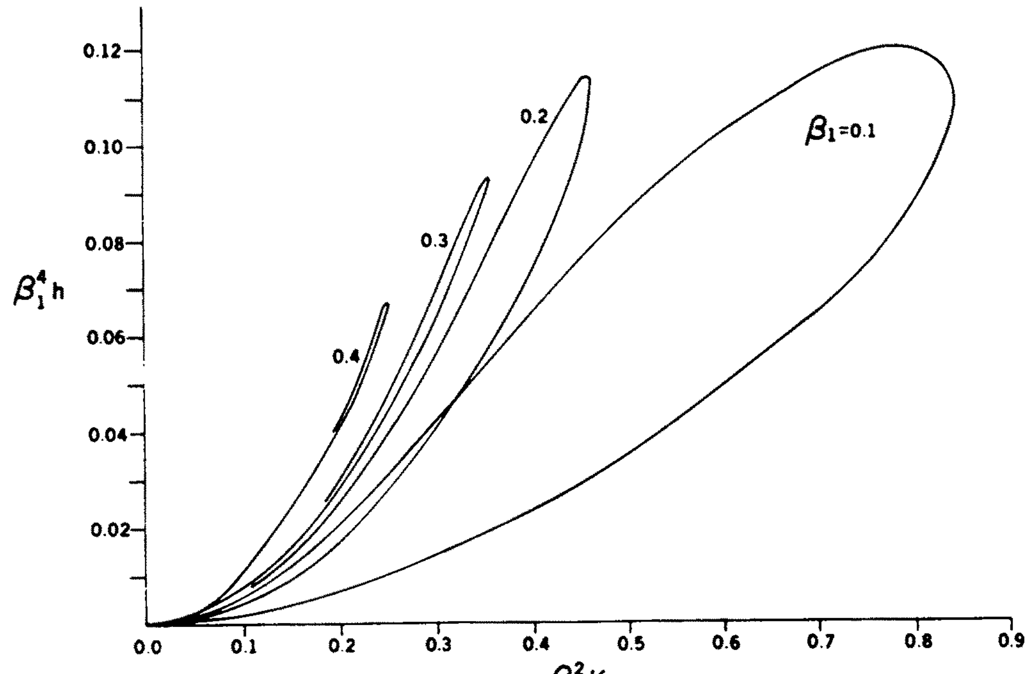

Fig. 6. Step height vs stress-intensity factor for two noncontract regions and various speeds.

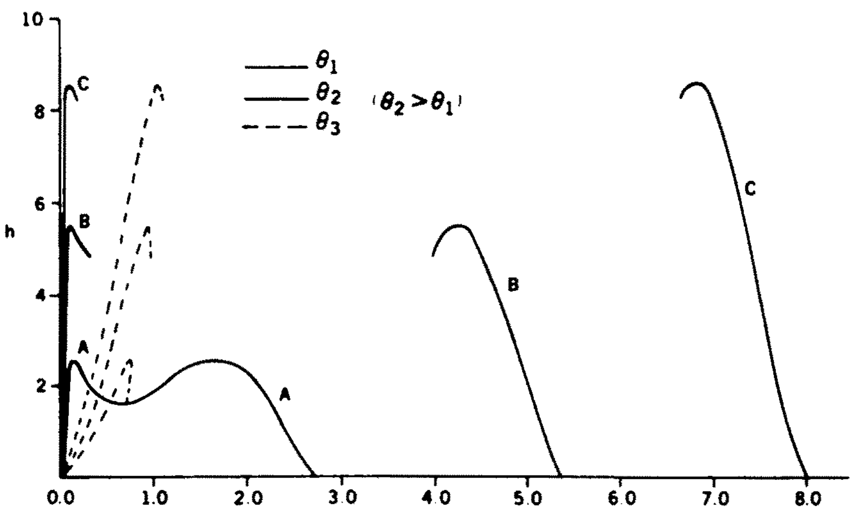

Fig. 7. Step height (h) vs noncontact lengths (θ_1, θ_2, θ_3) for fixed speed ($\beta_1 = 0.4$) and two noncontact regions.

usually correspond to greater values of stress-intensity factor. The problem is, of course, nonlinear due to the existence of noncontact regions.

In Fig. 7 we show h vs θ_1, θ_2 and θ_3 for the first three modes (A, B, C) corresponding to two noncontact regions at a fixed speed ($\beta_1 = 0.4$). Mode A begins with $\theta_1 = \theta_2$ and behaves as discussed in connection with Fig. 5. Mode B starts with one point on the lower boundary of the leading noncontact region touching the foundation. As θ_2 increases the point lifts off and h increases to a local maximum and then decreases to zero. This leaves mode B at the second free mode solution and a flat foundation. The behavior of mode C, as well as higher modes not shown, is qualitatively similar to that of B. As in [1–6] at given speed and step height an infinite number of solutions is possible. Due to the nonlinearity associated with the existence of noncontact regions, different initial conditions (which cause different transient solutions) can lead to different steady solutions. The results shown here can then be considered a collection of such solutions. Figure 8 shows step height (h) vs stress-intensity factor (K) for modes A, B, C

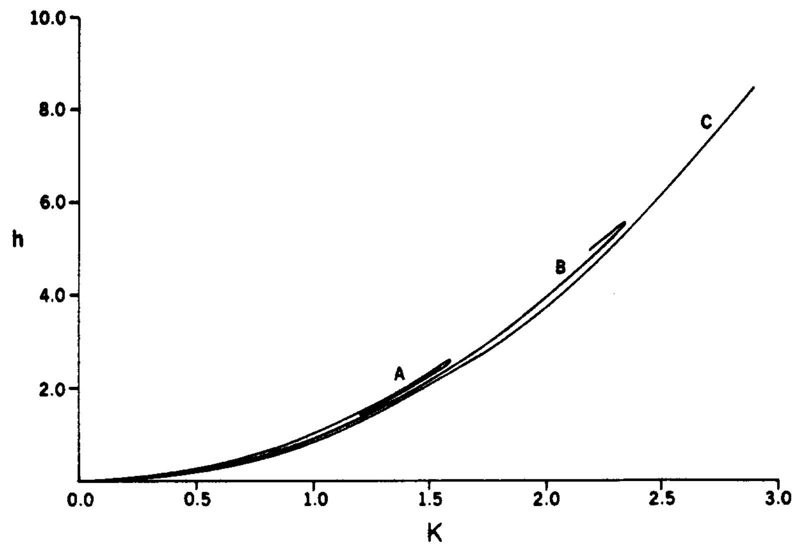

Fig. 8. Step height (h) vs stress-intensity factor (K) for fixed sneed ($\beta_1 = 0.4$) and two noncontact regions.

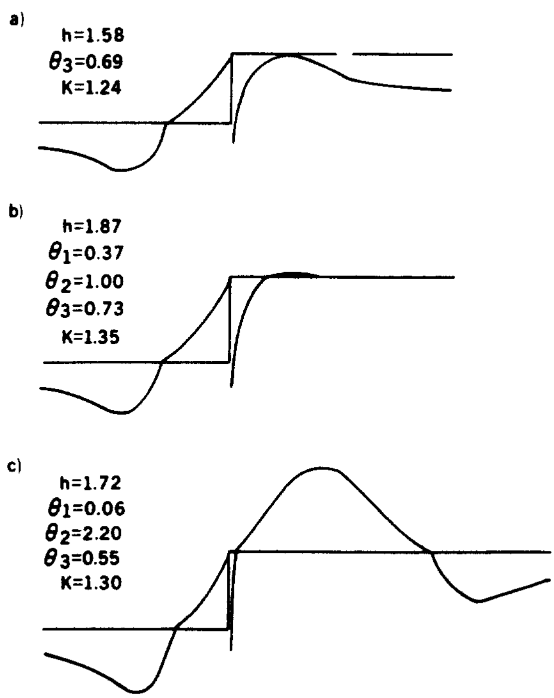

Fig. 9(a–c).

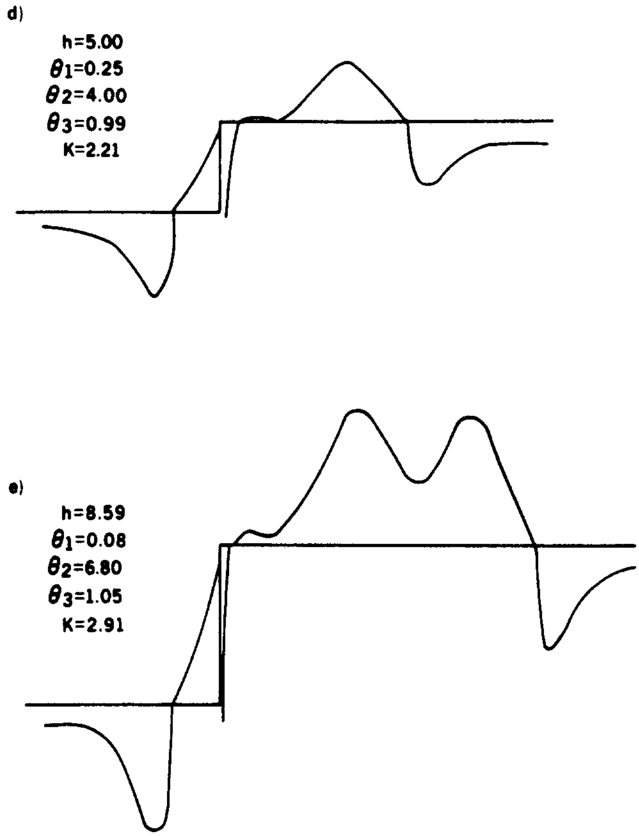

Fig. 9(a–e). Lower boundary displacement and foundation contact pressure vs ξ_1 for fixed speed ($\beta_1 = 0.4$).

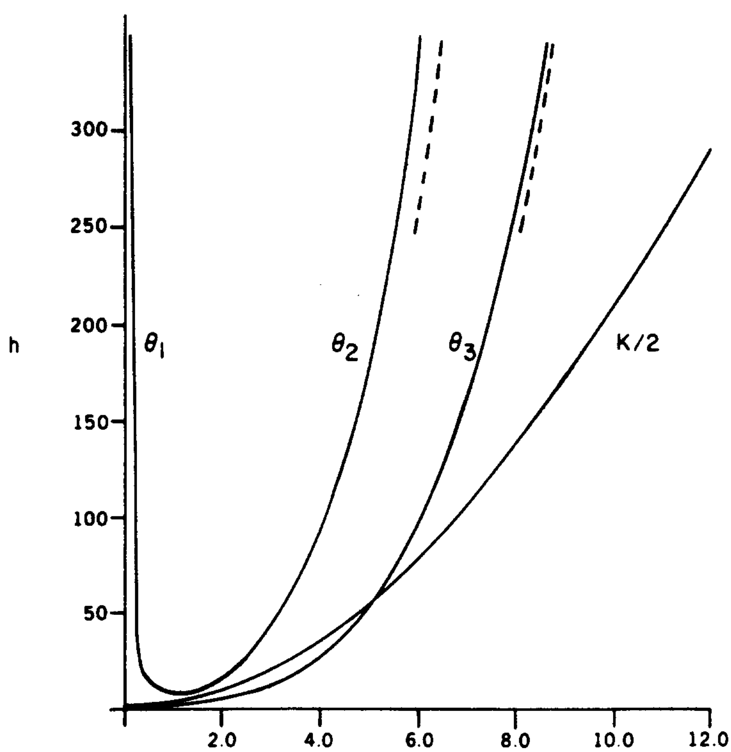

Fig. 10. Step height (h) vs noncontact lengths (θ_1, θ_2, θ_3) and stress-intensity factor (K) for a stationary strip. Dashed lines represent results from Euler-Bernoulli theory.

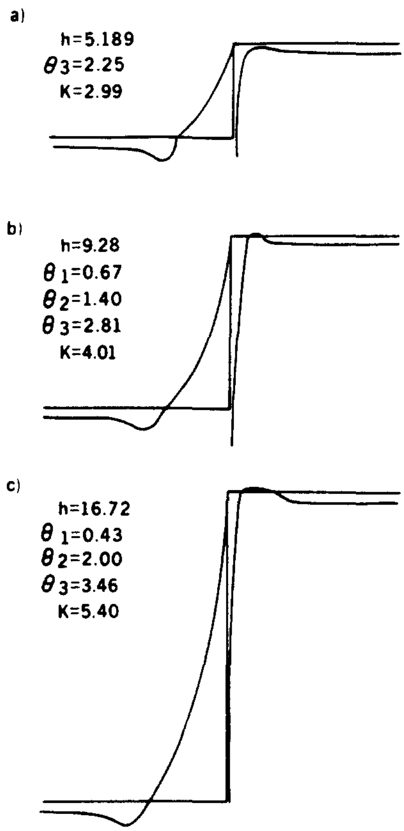

Fig. 11. Lower boundary displacement and foundation contact pressure vs ξ_1 for a stationary strip.

at fixed speed ($\beta_1 = 0.4$). Note that for given step height the value of K does not depend strongly on the mode. This is because the stress-intensity factor depends on the distances θ_3 and θ_1 to a much greater extent than it depends on θ_2. In Fig. 9(a–e) we show typical lower boundary displacements (plotted above the step profile) and foundation contact pressure (shown beneath the step) corresponding to the results shown in Fig. 7.

The corresponding results for a stationary strip are shown in Figs. 10 and 11 for one and two noncontact regions. Note that these solutions are unique and that θ_1, θ_2, θ_3 and K vary monotonically with step height h (Fig. 10).

Note that the results obtained for this plane strain analyses approach those determined using the simpler Euler–Bernoulli beam theory[6] as θ_3 increases. Recall that large values of the dimensionless θ_3 correspond to "thin" strips since θ_3 is the ratio of the significant length ($a\theta_3$) to the strip thickness (a). For small values of θ_3 the local effects such as the simularity at the step corner cause the two theories to give different results. Typical displacement shapes and foundation pressures are shown in Fig. 11(a–c).

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